Section 4-3, Mathematics 108

Functions

Some Definitions:

A **relation** is just a set of ordered pairs.

Example:

(1,4), (2,5), (3,6)

It maps some x values to y values, so we can draw it like this.

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

Note that you can also have a many to one relation

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

or one to many

$$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

Functions

A function is a special type of relation. What makes a relation different, is that each x only goes to one y.

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We use the script letter f to mean the function that takes elements from A to B

$$x \to f(x)$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

Note that each element x of A is mapped to exactly one element of B.

It is quite permissible for more than one element of A to be mapped to the same element of B. One to many is still allowed

$$x \to f(x)$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$$

The following relation is not a function, why?

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

The set of x values is known as the **domain** of the function.

The set of y values is known as the r**ange** of the function.

We call a variable such as x that represents an element of the domain the **independent variable.**

We call a variable that represents the y values the **dependent variable** because it depends on x.

So if we write

$$y = f(x)$$

x is the independent variable and y is the dependent variable.

Some Notes

1) The domain and range can be the same.

2) Two functions can have the same mapping but be different functions.

Example:

The **identity** mapping is a mapping that takes x to x .

Note that the identity mapping on the integers and the identity mapping on the real numbers are two different functions. A function consists of both the mapping and the domain.

3) We most often use the notation $f(x)$ to mean a function, but when we are dealing with more than one function, we may use $g(x)$, $h(x)$ or any other convenient letter or letters.

Ways to describe a function

1) Verbally

Example:
"The function which maps each student-id at USF to that particular Student".

In this case A is as set of student-id numbers and B is the set of students at USF.

2) Using a specific listing

x	$f(x)$
1	3
2	9
3	27

2) Using a table

Cost to send a 1st class letter in the US

ounces	price
≤ 1 ounce	.49
> 1 and ≤ 2 ounces	.70
> 2 and ≤ 3 ounces	.91

3) Using an algebraic expression

An example is a function that returns the area of a circle given it's radius.

$$A(r) = \pi r^2$$

Note that without explicitly knowing the domain of this function, we can assume it consists of all valid real numbers greater than 0.

This will be one of the most commonly used ways to describe a function

More on the domain of a function

Like the domain of an algebraic expression, the domain of a function may be stated explicitly, eg.

$f(x) = x^2 \quad 0 \le x \le 5$

If the domain is not otherwise stated, we assume it is a maximal subset of the real numbers.
That is the domain is the real numbers minus any values that are undefined.

Example:

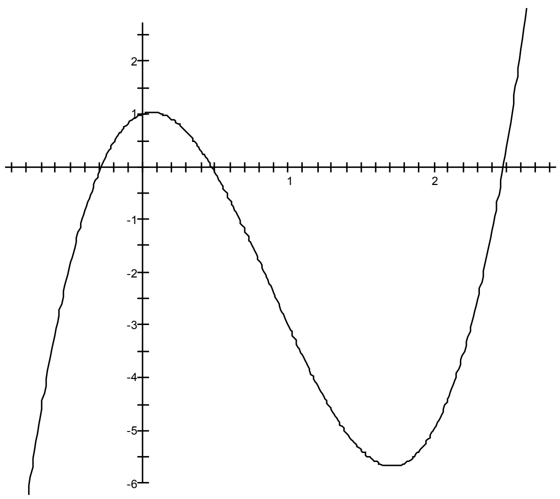
a) $f(x) = \dfrac{1}{x(x-1)}$ What is the domain?

b) $g(x) = \sqrt{9 - x^2}$ What is the domain?

Graphs of Functions, The Vertical Line Test

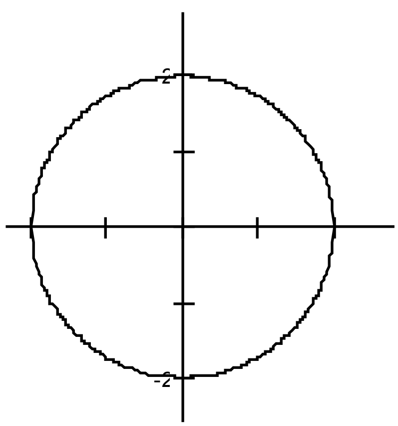
If you have the graph of a function, if you look at all possible vertical lines, if any of those lines pass through more than one point, then the graph is not a function. Why?

Example:



A function

Example:



Not a function

Evaluating a Function

To evaluate a function at some x value is to find the value of $f(x)$.

Example:

$f(x) = x^2 + 1$

$f(-2) = (-2)^2 + 1 = 5$

$f(0) = (0)^2 + 1 = 1$

Finding the Domain and Range of a Function

Example:
$f : \{(-3,0),(-1,2),(0,4),(2,4),(4,-1)\}$

The domain is clearly {-3,-1,0,2,4} and the range is clearly {0,2,4,-1}

Example:
Area of a square $A = s^2$

The length of a side has to be > 0 so the domain is s>0
Then the range is A>0

Looking at a graph: